methods at present exist, but the matter has been the subject of much calculation and difference of opinion. The ozone fortunately appears to be located somewhat as a layer in the vicinity of 50 kilometers. Thus, quite aside from the above interpretation of the ozone bands, any temperature effect on the absorption would afford a possible experimental means of determining the temperature of the atmosphere at the height of the ozone layer. Indeed it would be important to know of any effect, if such existed, since it would influence the determinations of the amount of ozone that is in the atmosphere. Since it is also important from the standpoint of the structure of the molecule and its photochemistry the influence of temperature upon the absorption is now being studied.

[1] Chappuis, *Ann. de l'ecole normale sup.*, [2] **11**, 137 (1882). *Compt. rend.*, **91**, 985 (1880); **94**, 858 (1882).

[2] Schoene, *J. Russ. Phys.-Chem. Soc.*, **16**, Part 9, p. 250(Dec. 20, 1884). *J. Chem. Soc.*, **48**, 713 (1884); *Chem. News*, **69**, 289 (1894).

[3] Ladenburg and Lehmann, *Ann. Phys.*, [4] **21**, 305 (1906). *Verh. d. deutsch. Phys. Ges.*, **8**, 125 (1906).

[4] Colange, *J. phys. et le rad.*, [6] **8**, 254 (1927).

[5] In thus arranging the bands it is taken that there are members of the strong progression at 6015 Å and 5650 Å and members of the weak progression at 6175 Å and 5790 Å. It will be noted that the band at 5650 Å is weaker than the first member at 6015 Å but stronger than the third member at 5335 Å.

[6] Warburg and Leithauser, *Ann. Phys.*, [4] **23**, 209 (1907).

[7] Chappuis (reference 1) reported a rather surprising influence of temperature upon the absorption of ozone which seems to have been somewhat overlooked, and which would appear to be something quite different from the effect suggested here.

ON THE ESTIMATION OF DISTANCES IN A CURVED UNIVERSE WITH A NON-STATIC LINE ELEMENT

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§1. *Introduction.*—In two previous articles,[1,2] I have shown that there is a possibility of relating the annihilation of matter in the universe to the observed red shift in the light from the extra-galactic nebulæ, by ascribing to the universe a line element of the general form

$$ds^2 = -\frac{e^{g(t)}}{\left[1 + \frac{r_2}{4R^2}\right]^2}(dx^2 + dy^2 + dz^2) + dt^2, \qquad (1)$$

where r is an abbreviation for $\sqrt{x^2 + y^2 + z^2}$ and R is a constant. This line element evidently corresponds to a spatially curved universe because of the term in r^2/R^2, and in addition is non-static because of the presence of the time in function $g(t)$.

The quantities x, y and z, occurring in equation (1) are, of course, merely spatial coördinates which can be thought of as laid off in a particular way that was convenient for the derivation of the line element in the form given. And the purpose of the present article is to exhibit, more fully than before, the relations between the distance r in these coördinates to a given nebula, and the actual observations of angular extension and luminosity that would be obtained by the astronomer for the purpose of estimating distances.

This is a matter of some importance since these relations are obviously essential for the theoretical interpretation of the actual astronomical observations. Moreover, it will be shown on the basis of the above line element that we should expect to find a definite relation between observations of angular extension and luminosity, which differs for example from that which we should expect for stationary objects in flat space-time, and since this difference appears to be just within the existing range of possible observational detection, the results of the present paper will afford the opportunity for a further check on the correctness of the proposed line element.

§2. *Transformation of Coördinates.*—As a preliminary to the discussion, it will first be desirable to give certain forms of the line element that result from changes to other systems of coördinates that sometimes prove useful.

Let us first change to "polar" coördinates by making the simple substitutions

$$x = r \sin \theta \cos \phi; \qquad y = r \sin \theta \sin \phi; \qquad z = r \cos \theta. \tag{2}$$

We can then evidently rewrite the line element (1) in the form

$$ds^2 = - \frac{e^{g(t)}}{\left[1 + \frac{r^2}{4R^2}\right]^2} (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2) + dt^2. \tag{3}$$

Let us now make a further change in our method of laying off the spatial mesh system by putting

$$\frac{r}{1 + \frac{r^2}{4R^2}} = \bar{r}. \tag{4}$$

It can then easily be shown that the line element assumes the form[3]

$$ds^2 = -e^{g(t)}\left(\frac{d\bar{r}^2}{1-\frac{\bar{r}^2}{R^2}} + \bar{r}^2 d\theta^2 + \bar{r}^2 \sin^2\theta d\phi^2\right) + dt^2. \qquad (5)$$

This form of the line element will prove to be the simplest to use in the actual deduction of expressions for angular extension and luminosity.

Finally, we shall change to a system of "rectangular" spatial coördinates by going to a space of higher number of dimensions. To do this we make use of the transformation equations

$$z_1 = R\sqrt{1-\bar{r}^2/R^2} \qquad z_2 = \bar{r}\sin\theta\cos\phi \qquad (6)$$
$$z_3 = \bar{r}\sin\theta\sin\phi \qquad z_4 = \bar{r}\cos\theta.$$

With these new variables it can be shown that the line element assumes the very simple form

$$ds^2 = -e^{g(t)}(dz_1^2 + dz_2^2 + dz_3^2 + dz_4^2) + dt^2. \qquad (7)$$

In using this form of the line element, however, it should not be forgotten that the actual space of the "world" occupies merely a three-dimensional "surface" in the four-dimensional manifold corresponding to $z_1 \ldots z_4$. By squaring the transformation equations (6) and adding we obtain as the equation of this "surface,"

$$z_1^2 + z_2^2 + z_3^2 + z_4^2 = R^2, \qquad (8)$$

and must remember that the physically significant values of $z_1 \ldots z_4$ will be limited by this equation. We shall later find use for this final form of the line element in determining the result of transferring the origin of spatial coördinates from one position to another.

§3. *The Motion of Particles and Light Rays.*—As a further preliminary to our discussion we need certain information as to the motion of particles and light rays that would agree with the above line elements.

Since the original line element was derived on the assumption that particles (nebulæ), which were stationary in the coördinates used, would remain permanently so, and since the transformation equations (2), (4) and (6) are independent of the time, it is evident that the nebulæ can also be taken as stationary in the coördinates used in the new forms of the line element given by equations (3), (5) and (7).

As to the behavior of particles, which are not stationary, we shall be especially interested in the motion of light quanta which are traveling radially to or from the origin of the coördinates used in forms (1), (3) and (5) of the line element. In general, of course, the motion of any particle will be determined by the equations for a geodesic

$$\frac{d^2x_\alpha}{ds^2} + \{\mu\nu,\alpha\}\frac{dx_\mu}{ds}\frac{dx_\nu}{ds} = 0. \qquad (9)$$

Applying this equation to a particle which is moving at a given instant in the x-direction along the x-axis in the original system of coördinates corresponding to (1), we evidently obtain for the acceleration in the y-direction

$$\frac{d^2y}{ds^2} + \{11,2\}\left(\frac{dx}{ds}\right)^2 + \{44,2\}\left(\frac{dt}{ds}\right)^2 = 0, \tag{10}$$

and introducing the expressions for the Christoffell symbols given by equations (19) in the article already mentioned,[4] this becomes

$$\frac{d^2y}{ds^2} - \frac{1}{2}\frac{\partial\mu}{\partial y}\left(\frac{dx}{ds}\right)^2 = 0, \tag{11}$$

where for abbreviation we have set

$$e^{\mu} = \frac{e^{g(t)}}{\left[1 + \dfrac{x^2 + y^2 + z^2}{4R^2}\right]^2} \tag{12}$$

as in the earlier article. And since for a particle on the x-axis the value of $\partial\mu/\partial y$ is evidently zero, we see that a particle once moving along the x-axis will remain permanently thereon.

Hence, since the x-axis can evidently be chosen so as to coincide with any radial direction of interest, we can conclude as is perhaps obvious from the spherical symmetry of the line element, that particles moving radially to or from the origin will not be deflected from that path. This result, which evidently applies in all three sets of coördinates (1), (3) and (5), and to light quanta as well as to material particles, will be of considerable use in what follows.

§4. *Transfer of Origin of Coördinates from Nebulæ to Observer.*—One, further preliminary result must still be obtained, which will be needed in our discussion of the luminosity of the nebulæ. Consider a system of coördinates S, corresponding to the line element in the form (5), and having a given nebula at the origin of coördinates and an observer located at the coördinate distance $\bar{r} = a$, which in accordance with the preceding section can be taken as a constant. The needed result now consists in showing the possibility of transforming to a new system of coördinates S', of the same form as before, but having the observer at the origin of coördinates and the nebula located at the coördinate distance $\bar{r}' = a$, the same numerical value as before.

To prove this result we shall not attempt to obtain the general transformation equations connecting the two systems of coördinates S and S', which turn out to be very complicated, but shall adopt a somewhat simpler procedure. In the original coördinate system S we shall take the coördinates for the nebula and the observer as shown in the following table.

System S	$\bar{r}$	θ	ϕ
Nebula	0	–	–
Observer	a	0	–

(13)

where for simplicity the observer is given the polar angle $\theta = 0$, since the starting points for measuring the angles θ and ϕ can evidently be chosen in any arbitrary way that proves convenient.

Let us now change to a new system of coördinates S_z which corresponds to the line element in the form (7). Making use of the transformation equations (6), we then easily obtain the following values for the new coördinates of the nebula and observer.

System S_z	z_1	z_2	z_3	z_4
Nebula	R	0	0	0
Observer	$R\sqrt{1 - a^2/R^2}$	0	0	a

(14)

Let us now consider a still further change of coördinates (rotation in the $z_1\, z_4$ plane) to a system of coördinates S_z', which we define by the transformation equations,

$$\begin{aligned} z_1' &= z_1 \cos\alpha + z_4 \sin\alpha \qquad & z_2' &= z_2 \\ z_4' &= -z_1 \sin\alpha + z_4 \cos\alpha \qquad & z_3' &= z_3 \end{aligned} \tag{15}$$

where we shall put

$$\sin\alpha = \frac{a}{R} \quad \text{and} \quad \cos\alpha = \sqrt{1 - \frac{a^2}{R^2}}. \tag{16}$$

Applying these equations we now obtain the following values for the coördinates of the nebula and observer.

System S_z'	z_1'	z_2'	z_3'	z_4'
Nebula	$R\sqrt{1 - a^2/R^2}$	0	0	$-a$
Observer	R	0	0	0

(17)

It will be noticed, however, from the form of the transformation equations (15) that this last transformation has been such as to leave the line element still in the form (7) and to preserve the relation (8) connecting the physically significant points in the 4-space $z_1' \ldots z_4'$. Hence, we may now again use transformation equations of the form (6) to get back to a system of polar coördinates S' of the original form (5), in which the nebula and observer will be found to have the coördinates

System S'	$\bar{r}'$	θ'	ϕ'
Nebula	a	π	—
Observer	0	—	—

(18)

Comparing tables (13) and (18), we see that we have now actually carried out the desired transformation from a system of polar coördinates with the nebula at the origin to one with the observer at the origin, and have proved the equality $\bar{r} = \bar{r}' = a$ of the respective radial coördinates of observer and nebula in the two systems. The result is perhaps an obvious consequence of the uniform distribution of matter assumed in the derivation of our original line element, but is so important for our later considerations as to justify the detailed proof that we have given.

§5. *Estimates of Distance from Observations of Angular Extension.*—We may now consider the estimation of distances from measurements of the mean angular extension of the nebulæ in a cluster. For the purposes of the discussion we shall take the line element in the form given by equation (5), and shall take the observer at the origin and the nebula to be observed as located at the coördinate distance $\bar{r}$, so that we may regard the light from the edges of the nebula as traveling radially in to the observer. Furthermore, we shall take t_1 as the time at which light leaves the nebula and t_2 as the time of its arrival at the origin when the observation is made.

Let us now take δs_0 as the diameter of the nebula as measured in proper coördinates by a local observer at the time t_1, and for convenience let us consider this diameter as lying in the direction of $\delta\theta$. From the form of the line element we can then evidently write as a relation connecting the proper diameter of the nebula δs_0 with the coördinate angle $\delta\theta$, the expression

$$\delta s_0 = e^{g_1/2}\, \bar{r}\delta\theta, \tag{19}$$

where g_1 denotes the value of $g(t)$ at the time t_1. Moreover, since we have shown above that light will travel radially toward the origin, it is also evident that $\delta\theta$ will be the angular extension ascribed to the nebula by the observer at the origin.

To use the above expression, we may compare this value for angular extension with that for a standard comparison nebula of the same intrinsic dimensions, which is located at a standard coördinate distance $\bar{r}_s$ and emits at time t_s light which reaches the observer at the same time t_2 as that from the nebula under observation. As an expression for the proper diameter of this standard nebula we can evidently write

$$\delta s_0 = e^{g_s/2}\,\bar{r}_s \delta\theta_s, \tag{20}$$

where g_s denotes the value of $g(t)$ at time t_s.

Equating the expressions for the proper diameters of the two nebulæ, given by equations (19) and (20), we can then write

$$\bar{r} = \bar{r}_s \frac{e^{-g_1/2}}{e^{-g_s/2}} \frac{\delta\theta_s}{\delta\theta} \tag{21}$$

as an expression connecting the coördinate distance $\bar{r}$ of the nebula under observation with the value $\bar{r}_s$ for the standard nebula, and the measured angular extensions $\delta\theta$ and $\delta\theta_s$ for the two nebulæ.

This result, however, can now be put in a more convenient form by introducing the expression previously obtained[5]

$$\frac{\lambda + \delta\lambda}{\lambda} = e^{\frac{g_2 - g_1}{2}} \tag{22}$$

which gives the red shift in the light from a nebula in terms of the values of $g(t)$ at the time of departure and arrival of the light. Multiplying both the numerator and denominator of the right-hand side of equation (21) by $e^{g_2/2}$ and introducing equation (22) we thus obtain

$$\bar{r} = \bar{r}_s \frac{(1 + \delta\lambda/\lambda)}{(1 + \delta\lambda_s/\lambda_s)} \frac{\delta\theta_s}{\delta\theta} \tag{23}$$

where $\delta\lambda/\lambda$ is the fractional red shift in the light from the nebula under observation and $\delta\lambda_s/\lambda_s$ is that for the comparison nebula.

Before proceeding, two criticisms of the above treatment must be mentioned. In the first place, the argument has been based on a line element which ignores the local gravitational field of the nebula under examination, and the light which is finally observed can suffer deflection in passing through this local field. In the second place, the nebula selected for examination and the comparison nebula emit the light which is finally observed at different times and no allowance has been made for possible difference in their dimensions due to difference in age. In first approximation, however, it is not thought that either of these difficulties is necessarily very serious, especially since in actual practice nebulæ which appear to be in the same evolutionary stage will be compared, with the result that they will tend to have the same proper dimensions and to produce the same deflections in light passing through their local fields.

§6. *Estimates of Distance from Observations of Luminosity.*—We now turn to the problem of estimating the distances of nebulæ from measurements of their luminosity. For the purposes of the discussion we shall again use the line element in the form given by equation (5), but in the first instance shall take the nebula under examination as located at the origin of coördinates and the observer as located at the coördinate distance $\bar{r}$, in order that we may regard the light as traveling radially out from the nebula.

As before, let us denote by t_1 the time at which light leaves the nebula and by t_2 the time at which it arrives at the observer. In addition let us take Z_0 as the rate at which light quanta of the mean energy E_0 are leaving the nebula, the measurements being made in proper coördinates which are stationary with respect to the nebula.

We must now calculate the value L which the observer will obtain for the luminosity of the nebula, which we define as the energy received from the nebula per unit time and per unit cross-section as measured in the local coördinates of the observer. To do this we note, from the form of the line element (5), that the proper area of the spherical surface at $\bar{r}$ through which and perpendicular to which the quanta are passing at time t_2 is evidently $4\pi\ \bar{r}^2\ e^{g_2}$; that the average energy of the quanta as measured at this surface will evidently be $E_0/(1 + \delta\lambda/\lambda)$; and finally in accordance with our previous work[5] that a short time interval dt_1 during which quanta leave the nebula will be connected with the time interval dt_2 in which they reach the surface at $\bar{r}$ by the relation

$$\frac{dt_2}{dt_1} = e^{\frac{g_2 - g_1}{2}} = 1 + \delta\lambda/\lambda. \tag{24}$$

With these considerations in mind we can then evidently write for the measured luminosity of the nebula

$$L = \frac{Z_0}{4\pi\bar{r}^2 e^{g_2}} \frac{E_0}{1 + \delta\lambda/\lambda} \frac{dt_1}{dt_2} = \frac{Z_0 E_0}{4\pi\bar{r}^2 e^{g_2}} \frac{1}{(1 + \delta\lambda/\lambda)^2}. \tag{25}$$

And for the luminosity of a similar standard comparison nebula we may write

$$L_s = \frac{Z_0 E_0}{4\pi\bar{r}_s^2 e^{g_2}} \frac{1}{(1 + \delta\lambda_s/\lambda_s)^2} \tag{26}$$

or combining the two equations we have

$$\bar{r} = \bar{r}_s \frac{(1 + \delta\lambda_s/\lambda_s)}{(1 + \delta\lambda/\lambda)} \sqrt{\frac{L_s}{L}}. \tag{27}$$

In accordance with the method of derivation the quantity r occurring in this equation locates the observer in a system of coördinates which was taken for convenience with the nebula at its origin. Nevertheless, since we have already shown in §4 that the radial coördinate for the nebula in a system of coördinates with the observer at the origin will have this same value, we can take equation (27), as locating the nebula in a system of coördinates with the observer at the origin.

Concerning the validity of equation (27), similar remarks can be made with regard to the neglect of local gravitational fields and the possible effects of a difference in age of the nebulæ as were made concerning equation (23). In addition it should be pointed out that the luminosities in equation (27) are obviously bolometric luminosities and corrections to pass from visual or photographic to bolometric magnitudes must be made if possible, before applying the equation to the usual astronomical observations.

§7. *Conclusion.*—The above equations connecting the coördinate distance $\bar{r}$ to the nebula with angular extension $\delta\theta$ and luminosity L were derived using the line element in the form (5), but with the help of the transformation equation (4) they can, of course, also be expressed in terms of the coördinate distance r occurring in the line element in the forms (1) and (3). For convenience we collect the results below

$$\bar{r} = \bar{r}_s \frac{(1 + \delta\lambda/\lambda)}{(1 + \delta\lambda_s/\lambda_s)} \frac{\delta\theta_s}{\delta\theta} = \bar{r}_s \frac{(1 + \delta\lambda_s/\lambda_s)}{(1 + \delta\lambda/\lambda)} \sqrt{\frac{L_s}{L}} \tag{28}$$

$$\begin{aligned} r &= r_s \frac{(1 + r^2/4R^2)}{(1 + r^2_s/4R^2)} \frac{(1 + \delta\lambda/\lambda)}{(1 + \delta\lambda_s/\lambda_s)} \frac{\delta\theta_s}{\delta\theta} \\ &= r_s \frac{(1 + r^2/4R^2)}{(1 + r^2/4R_s^2)} \frac{(1 + \delta\lambda_s/\lambda_s)}{(1 + \delta\lambda/\lambda)} \sqrt{\frac{L_s}{L}}. \end{aligned} \tag{29}$$

These are the equations to be used when we desire to obtain the coördinate distances of the nebulæ from the results of actual measurements of angular extension, luminosity and red shift.

Finally, it should be specially pointed out that the equality of the two different expressions for coördinate distance given by either (28) or (29) evidently leads to a relation between angular extension and luminosity

$$\frac{\delta\theta}{\sqrt{L}} = \text{const.} \times (1 + \delta\lambda/\lambda)^2 \tag{30}$$

which could be made the subject of experimental test. Furthermore, since the measurements have already been pushed to a distance where the value of the red shift $\delta\lambda/\lambda$ has risen to 0.04, it is hoped that this method of testing the theory may actually be possible in the not too distant future. The check on the theory thus provided would perhaps be a minor one, and the interpretation of the observations would be subject to the difficulties referred to at the end of §5 and of §6, but the undertaking would certainly be an interesting one since the relation, although agreeing with that which would be expected for objects in flat space-time with an actual motion of recession, differs from that which would be expected on the basis of some explanations of the red shift which might be proposed.

[1] Tolman, *Proc. Nat. Acad. Sci.*, **16**, 320 (1930).

[2] Tolman, *Ibid.*, **16**, 409 (1930).

[3] This is the form more similar to the usual Einstein one mentioned in my previous article (Ref. 1, p. 334) and the form of Robertson (see Ref. 2, § 2).

[4] See Ref. 1, p. 325.

[5] See Ref. 2, equation (21).

EXPERIMENTS IN BIRD MIGRATION. II. REVERSED MIGRATION

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The experiments in bird migration, which culminated during the winter 1929–1930 in an attempt to induce the common crow, *Corvus brachyrhynchos brachyrhynchos* Brehm, to migrate north instead of south in the fall, began in the autumn of 1924. The bird then used was the Junco, *Junco hyemalis connectens* Coues, a common breeder in central Alberta, available in large numbers, hardy and a fairly extensive migrant. The "working hypothesis" on which the experiments were based may be briefly outlined as follows. It was assumed that the migratory habit was, in the majority of migrants of the northern hemisphere, inherent and that food, cold and all the other factors, all of them highly variable, commonly held to account for the southward passage could not account for it except possibly in the case of certain races of a few species. Most species migrate with great regularity whatever the weather may do. In the case of the earlier migrants, leaving the north in July and early August, stress of circumstances can certainly not be cited as the stimulus to the southern urge. The accuracy from year to year of the dates of departure suggested that some environmental factor, which must be stable, was nevertheless involved as a timing agent. It was argued that this might well be found in the autumnal day-lengths. This, however, would not be sufficient to induce the passage if the tendency to undertake it were not inherent. An internal stimulus, affecting the physiological condition of the bird and arousing the impulse to depart also had to be stipulated. Internal secretions of the reproductive organs suggested themselves for the reason that the gonads of birds show a remarkable cycle coincident